Calculus Warm Up #4-3

New team # on your WU!

1. Find the net area and total area of the region between the curve and the x-axis.

$y = 2x^3 - 1$ on $[-1, 2]$

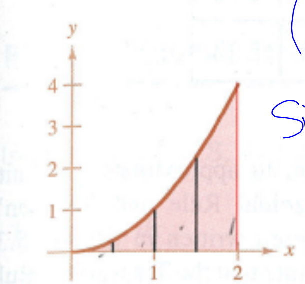
2. Find the approximate area by trapezoid rule and Simpson's rule. Compare both to the actual area.

$$\int_0^1 \sqrt{x+1}\,dx \quad , n = 4$$

HW Questions: p. 287

In Exercises 1–10, use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the indicated value of n. Round the answer to four decimal places and compare the results with the exact value of the definite integral.

1. $\int_0^2 x^2\,dx$



Trap ≈ 2.75

$\left(\frac{11}{4}\right)$

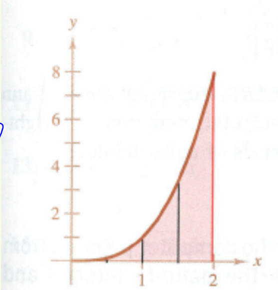
Simp ≈ 2.6667

$\frac{8}{3}$

Actual

$\frac{8}{3}$

3. $\int_0^2 x^3\,dx$



5. $\int_0^2 x^3\,dx,\ n = 8$

7. $\int_4^9 \sqrt{x}\,dx,\ n = 8$ $\quad h = \frac{5}{8} = 0.625$

9. $\int_1^2 \frac{1}{(x+1)^2}\,dx,\ n = 4$

11. $\int_0^4 \frac{1}{x+1}\,dx,\ n = 4$

$$A = \frac{5}{24}\Big[f(4) + \underline{4f(4.625)} + 2f(5.25) + \underline{4f(5.875)} + 2f(6.5) + \underline{4f(7.125)} + 2f(7.75) + \underline{4f(8.375)} + f(9)$$

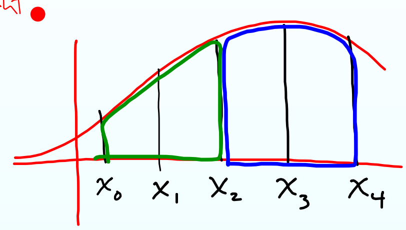
$$= \frac{5}{24}\Big[5 + 4\left(\sqrt{4.625} + \sqrt{5.875} + \sqrt{7.125} + \sqrt{8.375}\right) + 2($$

Using Simpson's Rule:

n= even # of subintervals

every 2 will make up a parabola region with a mid-height

Let n = 4

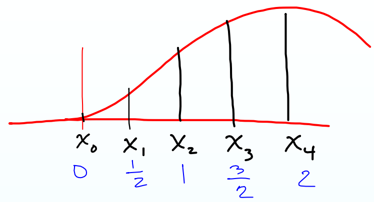


$A = \frac{h}{3}(\ell + 4m + r)$

$$A = \frac{h}{3}\Big[f(x_0) + 4f(x_1) + \underbrace{f(x_2) + f(x_2)}_{2f(x_2)} + 4f(x_3) + f(x_4)\Big]$$

Let n = 4

$$\int_0^2 5x^4\,dx = \boxed{32}$$



$f(x)=5x^4$

$$h = \frac{b-a}{n} \qquad h = \frac{2-0}{4} = \frac{1}{2}$$

$$\frac{h}{3} = \frac{1}{6}$$

$$A = \frac{1}{6}\left[f(0) + 4f\left(\tfrac{1}{2}\right) + 2f(1) + 4f\left(\tfrac{3}{2}\right) + f(2)\right]$$

$$= \frac{1}{6}\left[0 + \cancel{4}\left(\frac{5}{\cancel{16}_{4}}\right) + 2(5) + 4\left(\frac{405}{16}\right) + 80\right]$$

$$= \frac{1}{6}\left[\frac{5}{4} + 90 + \frac{405}{4}\right]$$

$$\approx 32.083$$

Chapter 5 Review

The Fundamental Theorem of Calculus

Essentially: Differentiation and Definite Integration are inverse operations.

If *f* is continuous on [a, b], and F'(x) = f(x)

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

The Second Fundamental Theorem of Calculus

Fomalizing the idea that differentiation and integration are inverse operations.

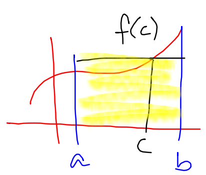
$$f(x) = \frac{d}{dx}\int_a^x f(t)\,dt$$

EXAMPLE 6 Applying the Second Fundamental Theorem of Calculus

Evaluate

$$\frac{d}{dx}\int_0^x \sqrt{t^2+1}\,dt. = \sqrt{x^2+1}$$

MVT for Integrals:



$$\underbrace{(f(c))(b-a)}_{\text{(height)(width)}}^{\text{rectangle}} = \int_a^b f(x)\,dx$$

The value of $f(c)$, given in the Mean Value Theorem for Integrals, is called the **average value** of f on the interval $[a, b]$.

Definition of the Average Value (outcome) of a Function on an interval.

If f(x) is integrable on [a, b], then

f(c) = average value

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Classwork: Ch. 5 Review (pink)

Classwork - review Ch. 5 Name____________ Team____

Find the indefinite integrals Per___

1. $\int \frac{2}{\sqrt[3]{3x}}\,dx$

2. $\int \frac{x^3 - 2x^2 + 1}{x^2}\,dx$

3. $\int x^2\sqrt{x^3 + 3}\,dx$

4. $\int \frac{x^2 + 2x}{(x+1)^2}\,dx$

Find the definite integrals

5. $\int_{-1}^{1} (t^2 + 2)\, dt$

6. $\int_{3}^{6} \frac{x}{3\sqrt{x^2 - 8}}\, dx$

7. $\int_{0}^{1} x^2(x^3 + 1)^3\, dx$

8. $\int_{-1}^{\sqrt[3]{17}} \frac{x^2}{x^3 - 1}\, dx$

9. A function f has a second derivative $f''(x) = 6(x - 1)$. Find the function if its graph passes through the point $(2, 1)$ and at that point is tangent to the line given by $3x - y - 5 = 0$.

10. Find the c guaranteed by The Mean Value Theorem and the average value of the given function for the given interval.

$f(x) = x^3$ $\quad\quad$ $[0, 2]$

HW: p. 289 # 1-17 odd,

21 - 33 odd, 45, 47, 49